NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science Honours in Applied Statistics	
QUALIFICATION CODE: 08BSHS	**LEVEL:** 8
COURSE CODE: STP801S	**COURSE NAME:** STOCHASTIC PROCESSES
SESSION: JUNE, 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Prof Rakesh Kumar
MODERATOR:	Prof Lawrence Kazembe

INSTRUCTIONS

1. Attempt any FIVE questions. Each question carries equal marks.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Question 1. (Total Marks: 20)

(a) What is a stochastic process? Give one example of a stochastic process. (7 marks)

(b) A particle performs a random walk with absorbing barriers, say 0 and 4. Whenever it is at position r ($0<r<4$), it moves to r+1 with probability p or to r-1 with probability q, p+q=1. But as soon as it reaches 0 or 4, it remains there. The movement of the particle forms a Markov chain. Write the transition probability matrix of this Markov chain. (7 marks)

(c) Differentiate between sub-martingale and super-martingale. (6 marks)

Question 2. (Total marks: 20)

(a) Show that the transition probability matrix along with the initial distribution completely specifies the probability distribution of a discrete-time Markov chain. (10 marks)

(b) Derive the Chapman-Kolmogorov equations for continuous-time Markov chain. (10 marks)

Question 3. (Total marks: 20)

Classify the states of the Markov chain whose transition probability matrix is given below:

$$\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(20 marks)

Question 4. (Total marks: 20)

Let N(t) be a Poisson process with rate $\lambda > 0$. Prove that the probability of n occurrences by time t, $P_n(t)$ is given by

$$P_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}; n = 0,1,2,3, \dots$$

(20 marks)

Question 5. (Total marks: 20)

Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is 1/3 and that probability of a rainy day following a dry day is 1/2. Develop a two-state transition probability matrix of the Markov chain. Given that April 21, 2023 is a dry day, find the probability that April 23, 2023 is a dry day.

(20 marks)

Question 6. (Total marks: 20)

(a) What is a Poisson process? (5 marks)

(b) Derive the steady-state probability distribution of birth-death process. (15 marks)

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